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Technical Notes

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Advanced Algebraic Model for Turbulent Diffusion Vector in Two-Equation Turbulence Models

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I. Introduction

TWO-EQUATION turbulence models are powerful tools for predicting various turbulent flows encountered in engineering applications. In calculating turbulent flows with two-equation models, the k–ε model is most commonly used, where k is the turbulent energy and ε is the dissipation rate of k. The transport equation of k includes the turbulent diffusion term, which can have a great influence on the prediction accuracy for turbulent flows far from the homogeneous state. To model the turbulent diffusion term properly, it is necessary to reproduce the direction of the following turbulent diffusion vector:

$$\overline{\left(\frac{uu+vv+ww}{2}\right)u_i} = \overline{k'u_i} \tag{1}$$

where u_i is the velocity fluctuation in the i direction and $\overline{k'} = k$. Thus far, two-equation models usually have adopted a simple isotropic eddy-viscosity expression for the Reynolds stresses ($\overline{u_iu_j}$), and, thus, the direction of the turbulent diffusion vector in Eq. (1) has not been discussed much, in contrast to the field of the full Reynolds stress model.[1–7]

Recently, however, nonlinear eddy-viscosity models have made great progress in predicting anisotropy of the Reynolds-stress components, and, consequently, higher-order algebraic models for turbulent diffusion have been adopted in some cases.[8,9] Therefore, a discussion of the turbulent diffusion models is of increasing importance, even in the category of the two-equation model. The most representative algebraic expression taking account the stress anisotropy is based on the generalized gradient-diffusion hypothesis (GGDH).[1] Although the GGDH model is practically useful, some major problems still remain. A crucial problem noted is that it gives an extreme underprediction of the streamwise component compared with that in the wall-normal direction when it is applied to wall-bounded turbulent shear flows. In previous studies[10,11] of the turbulent scalar transfer, Abe and Suga[10] and Suga and Abe[11] examined the relation between the scalar-flux vector and the Reynolds-stress tensor by using large eddy simulation (LES) data[10] and then proposed a new way of modeling the turbulent scalar fluxes.[11] Abe and Suga[10] also pointed out that the same modeling concept might have the capability for predicting the direction of the turbulent diffusion vector more precisely.

The final goal of the present study is to propose a new algebraic model for the turbulent diffusion vector expressed by Eq. (1). As the first step in this study, some representative algebraic expressions are reexamined carefully, with a focus mainly on the direction of the predicted turbulent diffusion vector. To reveal the characteristics and performance of the model, a priori tests are carried out by processing the LES data of a fundamental channel flow.[10]

II. Relation Between Turbulent Diffusion Vector and Reynolds Stress Tensor

First, we consider the following conventional eddy-viscosity model (EVM):

$$\overline{k'u_i} = -C_k k\tau_d \frac{\partial k}{\partial x_i}\left(\rightarrow = -C_k k\tau_d \delta_{ij}\frac{\partial k}{\partial x_j}\right) \tag{2}$$

where τ_d is the characteristic timescale and C_k is the model coefficient. In a case where the spatial gradient of k exists only in the y (wall-normal) direction, the turbulent diffusion vector is expressed by the EVM as follows:

$$\overline{k'u} = 0, \qquad \overline{k'v} = -C_k k\tau_d \frac{\partial k}{\partial y} \tag{3}$$

The representative expression of the GGDH model[1] is generally

$$\overline{k'u_i} = -C_k\overline{u_iu_j}\tau_d\frac{\partial k}{\partial x_j}\left(\rightarrow = -C_k k\tau_d\frac{\overline{u_iu_j}}{k}\frac{\partial k}{\partial x_j}\right) \tag{4}$$

In a case where only $\partial k/\partial y$ exists, the GGDH model gives the following expressions for the components of the turbulent diffusion vector:

$$\overline{k'u} = -C_k k\tau_d\frac{\overline{uv}}{k}\frac{\partial k}{\partial y}, \qquad \overline{k'v} = -C_k k\tau_d\frac{\overline{vv}}{k}\frac{\partial k}{\partial y} \tag{5}$$

On the other hand, it was indicated from the previous study[10] that the introduction of the quadratic products of $\overline{u_iu_j}$ into the model expression is expected to be useful in predicting the direction of the turbulent diffusion vector more precisely. The expression may be written as

$$\overline{k'u_i} = -C_k k\tau_d\frac{\overline{u_iu_l}}{k}\frac{\overline{u_lu_j}}{k}\frac{\partial k}{\partial x_j} \tag{6}$$

In this study, Eq. (6) is referred to as the quadratic model. When the flowfield is two dimensional and only $\partial k/\partial y$ exists, the quadratic model gives the turbulent diffusion vector as

$$\overline{k'u} = -C_k k\tau_d\left(\frac{\overline{uu}}{k}\frac{\overline{uv}}{k} + \frac{\overline{uv}}{k}\frac{\overline{vv}}{k}\right)\frac{\partial k}{\partial y}$$

$$\overline{k'v} = -C_k k\tau_d\left(\frac{\overline{vu}}{k}\frac{\overline{uv}}{k} + \frac{\overline{vv}}{k}\frac{\overline{vv}}{k}\right)\frac{\partial k}{\partial y} \tag{7}$$

The primary concern is the direction of the turbulent diffusion vector predicted by the models. As an estimation, distributions of C_k evaluated by the LES data[10] for these three models are shown in Fig. 1, together with the profiles of the turbulent diffusion vector in Eq. (1). The LES data used were obtained for a fundamental channel flow of $Re_\tau = u_\tau\delta/\nu = 180$, where u_τ is the friction velocity, δ is the half-width of the channel, and ν is the kinematic viscosity. Their reliability was suitably confirmed in the previous study.[10] Note that some discontinuities are seen in the C_k distributions of Fig. 1. They correspond, however, to the location of $\partial k/\partial y = 0$, and, thus, it is

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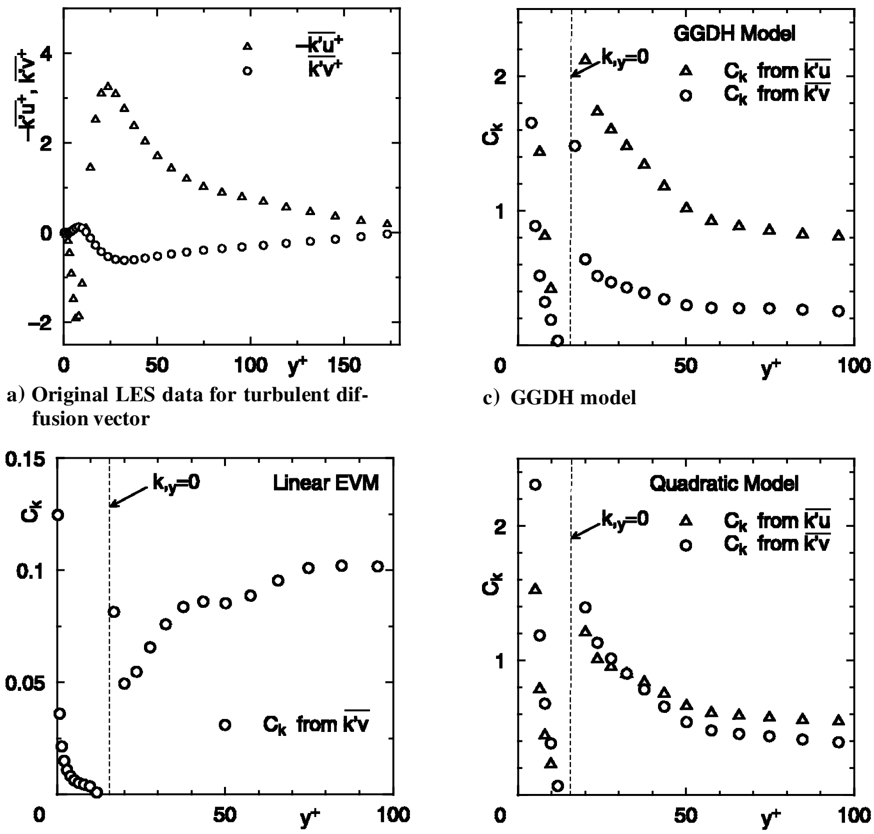


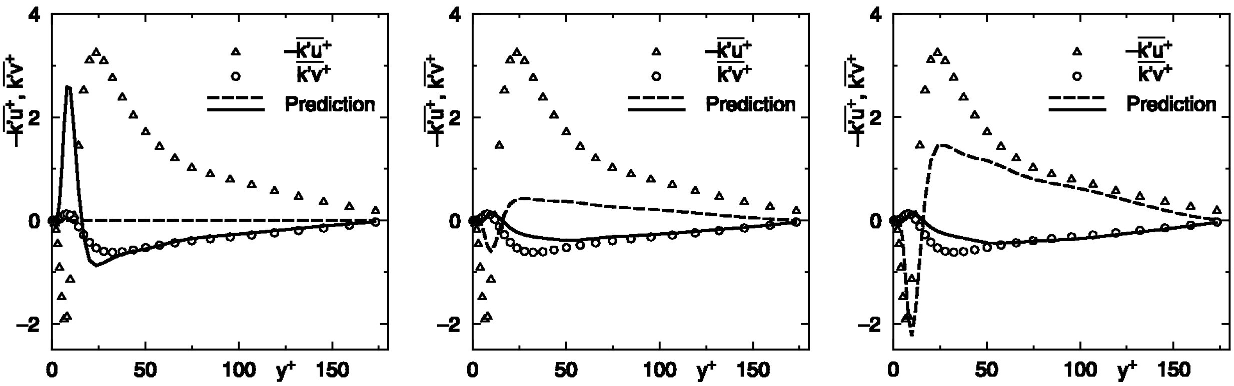
a) Original LES data for turbulent diffusion vector

b) C_k estimated by the EVM

c) GGDH model

d) Quadratic model

Fig. 1 Estimation of C_k for various model formulas.



a) EVM ($C_k = 0.09$)

b) GGDH model ($C_k = 0.22$)

c) Quadratic model ($C_k = 0.45$)

Fig. 2 Estimation of basic predictive performance.

not necessary to focus on this in the following discussion. In the estimations, τ_d is defined as k/ε and C_k is calculated in the x and y directions, respectively. For example, in the case of the quadratic model, C_k is estimated by

$$C_k = -\overline{k'u_i} \Bigg/ k\frac{k}{\varepsilon}\frac{\overline{u_i u_l}}{k}\frac{\overline{u_l v}}{k}\frac{\partial k}{\partial y} \qquad (i = 1, 2) \tag{8}$$

As seen in Fig. 1b concerning the EVM, C_k for $\overline{k'v}$ that is in the y direction shows a constant level of around 1 in the region far from the wall, which is consistent with generally accepted values. As for the GGDH model shown in Fig. 1c, the evaluated C_k for $\overline{k'v}$ shows a level of around 0.2–0.3, which is also consistent with generally accepted values. However, C_k for $\overline{k'u}$ is about three times as large as that for $\overline{k'v}$. This indicates that it is impossible to satisfactorily predict both components of Eq. (1) by using the GGDH model.

In contrast to this, as seen in Fig. 1d, distributions of C_k in both the x and y directions show nearly the same tendency. This is a notable feature, and it indicates that the direction of the turbulent diffusion vector can be predicted by the quadratic model much more precisely for this case.

III. Estimation of Basic Predictive Performance of the Models

In this section, we try to examine the basic performance of the models just described. To focus the discussion on the primary concern in this study, a suitable constant value of the coefficient C_k was employed for Eqs. (2), (4), and (6), respectively. As for the EVM [Eq. (2)] and the GGDH model [Eq. (4)], we adopted $C_k = 0.09$ and 0.22, respectively, following the generally accepted values. Concerning the quadratic model, Fig. 1d suggests that the model coefficient C_k is around 0.5 in the region away from the wall; hence, $C_k = 0.45$ was employed in this study in the first attempt.

In what follows, the predictive performance of the EVM, the GGDH, and the quadratic models is estimated by a priori tests using the aforementioned LES data. Distributions of the turbulent diffusion vector predicted by the three models are compared with the original LES data in Fig. 2. As seen in Fig. 2, the wall-normal component ($\overline{k'v}$) is generally predicted well in the region far from the wall by all three models, although the EVM provides excessive overprediction close to the wall ($y^+ \sim 10$).

In contrast to this, the predictive performance of $\overline{k'u}$ (in the x direction) shows significant differences between the models. Because the turbulent diffusion vector predicted by the EVM completely aligns with the spatial gradient of k, the EVM cannot predict $\overline{k'u}$ as shown in Fig. 2a [see Eq. (3)]. The GGDH model has some possibility of predicting the streamwise component by introducing the Reynolds-stress anisotropy. As seen in Fig. 2b, however, the GGDH model gives extreme underprediction of $\overline{k'u}$ and, thus, its predictive performance still leaves quite a wide margin for improvement.

On the other hand, the quadratic model generally gives a reasonable prediction for both $\overline{k'u}$ and $\overline{k'v}$, although some underprediction is seen in the limited region around $y^+ \sim 30$. Note that no previously proposed explicit algebraic model for Eq. (1) could capture this basic feature of $\overline{k'u}$ as precisely as the present quadratic model does. Hence, it is expected that the quadratic model will lead to more advanced algebraic turbulent diffusion models with further modification for the model coefficient C_k.

IV. Conclusions

To improve the prediction accuracy for the turbulent diffusion term appearing in two-equation models, a priori estimations have been performed by processing the LES data of a fundamental channel flow. In this study, some representative algebraic expressions were reexamined carefully, and we proposed a new way of modeling the turbulent diffusion. It introduces the quadratic products of the Reynolds-stress tensor, appropriately reflecting the knowledge obtained from the present investigation. A priori tests show that the model has the basic capability of predicting the turbulent diffusion term much more precisely, not only in the wall-normal direction, but also in the streamwise direction.

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Parallel Performance Analysis of FVTD Computational Electromagnetics Code

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Introduction

COMPUTATIONAL processing capability continues to limit the complexity and size of numerical simulations. Advancements in computer hardware technology, notably the development of powerful reduced instruction set computing microprocessors, high-density dynamic random access memory, and ultrahigh-speed switching networks now allow the construction of machines with the power to run applications across hundreds to thousands of processors that outstrip the performance of traditional vector supercomputers. This advancement in hardware still requires added effort in algorithm design because intelligent compilers for these machines do not yet fully exploit the parallelism in a computer code or map that parallelism to a distributed memory environment. It remains, therefore, a crucial task to design and test software for the best possible parallel performance across a variety of machines. At present, the popular practice of domain decomposition achieves a high degree of parallelism in grid-based engineering applications of computational electromagnetics (CEM) and computational fluid dynamics (CFD).

Because the Maxwell equations form a set of hyperbolic partial differential equations, like the equations of fluid dynamics, methods developed in CFD apply also to CEM. The finite difference time-domain approach on staggered grids was pioneered by Yee[1] in 1966, and Shankar et al.[2] developed a finite volume approach with the method originally developed by Lax and Wendroff.[3] Of interest also is the technique introduced by Shang[4] incorporating the flux-vector splitting methodology of Steger and Warming,[5] well known in CFD. This technique has been implemented and tested against theoretical solutions, experimental data, and frequency-domain methods.[6] To continue the development of this technique with parallel machines, the finite volume time-domain electromagnetic computer code CHARGE was developed by Blake[7] at Wright–Patterson Air Force Base. A domain decomposition strategy was also implemented and analyzed by Blake and Buter,[8] who concluded that although theory favored higher-dimensional decompositions for superior performance (in terms of scalability), this did not always result in practice. That work also indicated that parallel performance improved when the dimensionality of the domain decomposition closely matched the physical topology of the underlying machine architecture. One question that remained was whether an automatic domain decomposition strategy could evenly divide the work among a given number of processors (load balancing) regardless of machine architecture (portability). Rapid advances in computer hardware also required that the electromagnetic computer code again be compiled and tested on a variety of platforms. This study, therefore, focused on the portability and scalability of the CEM code on the massively parallel machines currently available.

Theoretical Background

Maxwell equations are well known to describe adequately electromagnetic phenomena of engineering interest. Constitutive relations for simple media (linear, isotropic, and homogeneous) include $\boldsymbol{D} = \epsilon \boldsymbol{E}$, $\boldsymbol{B} = \mu \boldsymbol{H}$, and $\boldsymbol{J} = \sigma \boldsymbol{E}$. These allow some flexibility in

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